

Self-accelerated Universe¹

B. P. Kosyakov

Institute for Theoretical and Mathematical Physics,

Russian Federal Nuclear Center–VNIIEF, Sarov 607190, Russia

E-mail address: kosyakov@vniief.ru

Abstract

It is widely believed that the large redshifts for distant supernovae are due to the vacuum energy dominance, or, more precisely, due to a cosmological constant in Einstein's equations, which is responsible for the anti-gravitation effect. A tacit assumption is that particles move along geodesics for the background metric. This is in the same spirit as the consensus regarding the uniform Galilean motion of a free electron. We note, however, that, apart from the Galilean solution, there is a self-accelerated solution to the Abraham–Lorentz–Dirac equation governing the behavior of a radiating electron. Likewise, a self-accelerated solution to the entire system of equations, both gravitation and matter equations of motion including, may exist, which provides an alternative explanation for the accelerated expansion of the Universe, without recourse to the hypothetic cosmological constant.

1 Introduction

The recent measurements of redshifts for Type Ia supernovae [1] suggest that the Universe expansion is accelerating. To interpret this discovery, one usually write the Friedmann equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{2}{A_V^2} + \frac{2A_D}{a^3} + \frac{2A_B}{a^3} + \frac{A_R}{a^4} - k \quad (1)$$

where H is the Hubble expansion parameter, a is the scale factor, A_V , A_D , A_B , and A_R are Friedmann integrals of the motion related to the energy density of vacuum, dark matter, nonrelativistic particles (baryons), and radiation; k is the spatial curvature, with $k = 1, -1, 0$ corresponding to the closed, open, and flat models. Equation (1) is derived from Einstein's equations with a positive cosmological constant

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad (2)$$

using the generic form of the line element for homogeneous and isotropic spacetimes

$$ds^2 = dt^2 - a^2 F(r)^2 d\Omega^2 - a^2 dr^2. \quad (3)$$

Here, t is the proper time, $d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$, $F = \sin r$, $\sinh r$, r for $k = 1, -1, 0$, respectively, the cosmological constant Λ relates to the vacuum energy density ρ_V as $\Lambda = 8\pi G \rho_V$, and $A_V = (8\pi G \rho_V / 3)^{-1/2}$. A pseudo-Riemannian space with arbitrary curvilinear coordinates x^μ and metric tensor $g_{\mu\nu}$ is understood in this cosmological model.

In the expanding Universe, the scale factor a increases with time. So, there comes a time when the first term in (1) becomes dominant. The asymptotic ($t \rightarrow \infty$) solution to Eq. (1) is

$$a(t) = A_V f(t), \quad f(t) = \begin{cases} \cosh(t/A_V) & k = 1 \\ \sinh(t/A_V) & k = -1, \\ \exp(t/A_V) & k = 0 \end{cases} \quad (4)$$

¹An extended version of the author's talk given at the *9th Adriatic Meeting: Particle Physics and the Universe*, held in Dubrovnik, Croatia, 4–14 September 2003

which shows that the cosmological expansion accelerates, $\ddot{a} > 0$.

Thus, the presence of a positive cosmological constant Λ in (2), which is responsible for the anti-gravitation effect, ensures the accelerated expansion of the Universe. At present, this explanation of the large redshift data for distant supernovae is widely accepted (for a recent review see [2]) with the tacit belief that particles (galaxies, clusters, etc.) move along geodesics for the background metric $g_{\mu\nu}$. To impeach this belief, we first remark that galaxies and clusters every so often have intrinsic angular momentum, spin. It is well known, however, that a spinning particle is deflected from the geodesic [3]–[5]. Note that such an anomalous motion is unrelated to the spacetime geometry: a spinning particle in Minkowski space behaves in a non-Galilean manner. Indeed, let us consider a classical spinning particle in the Frenkel model [6]. Its motion, in the absence of external forces, is governed by the equation (see, e. g., [7])²

$$\mathcal{S}^2 \ddot{v}^\mu + M^2 v^\mu = m p^\mu \quad (5)$$

where \mathcal{S} is the spin magnitude, $v^\mu = \dot{z}^\mu$ is the four-velocity, the dot denotes the derivative with respect to the proper time s , p^μ is the four-momentum (which is constant for the free particle), M and m are the mass and rest mass, defined as $M^2 = p^2$ and $m = p \cdot v$. For $p^2 > 0$, $p^\mu = \text{const}$, the general solution to Eq. (5) is

$$z^\mu(s) = \frac{m}{M^2} p^\mu s + \frac{\alpha^\mu}{\omega} \cos \omega s + \frac{\beta^\mu}{\omega} \sin \omega s \quad (6)$$

where $\alpha \cdot p = \beta \cdot p = \alpha \cdot \beta = 0$, $\alpha^2 = \beta^2$, and $\omega = M/\mathcal{S}$. This helical world line describes motion called the Zitterbewegung (this phenomenon was first discovered by Schrödinger [8] in a quantum context). For $p^2 < 0$, $p^\mu = \text{const}$, we have

$$z^\mu(s) = -\frac{m}{\mathcal{M}^2} p^\mu s + \frac{\alpha^\mu}{\Omega} \cosh \Omega s + \frac{\beta^\mu}{\Omega} \sinh \Omega s \quad (7)$$

where $\mathcal{M}^2 = -p^2$, $\Omega = \mathcal{M}/\mathcal{S}$, and $\alpha^2 = -\beta^2$. This solution describes motion with increasing velocity.

One may argue that spacelike four-momenta p^μ are highly unnatural for classical particles. While this is a strong objection, it seems reasonable to say that both solutions (6) and (7) support the idea of non-Galilean regimes for free spinning particles.

Another fact deserving of notice is that massive particles can emit gravitational waves. It is conceivable that a massive particle emitting gravitational waves moves in a runaway regime, that is, deviates sharply from a geodesic for the background metric. It seems plausible that runaway solutions may offer an alternative explanation for the accelerated expansion of the Universe, without recourse to the cosmological constant hypothesis. If this explanation is true, one would gain the most benefit from it looking at a theoretical framework where $\Lambda = 0$, e. g., unbroken supersymmetry, for clues of the mystery of the cosmological constant [2].

It is interesting to compare a massive particle emitting gravitational waves and a charged particle emitting electromagnetic waves. The nonrelativistic equation of motion for a classical electron, called the Abraham–Lorentz equation (see, e. g., [9]–[12]),

$$m \mathbf{a} - \frac{2}{3} e^2 \frac{d\mathbf{a}}{dt} = \mathbf{f}, \quad (8)$$

in the absence of external forces $\mathbf{f} = 0$, becomes

$$\mathbf{a} - \tau_0 \frac{d\mathbf{a}}{dt} = 0 \quad (9)$$

where

$$\tau_0 = \frac{2e^2}{3m} \approx 6 \cdot 10^{-24} \text{ s}. \quad (10)$$

The general solution to Eq. (9),

$$\mathbf{a}(t) = \mathbf{A} \exp(t/\tau_0), \quad (11)$$

²Throughout this paper we use the Minkowski metric with signature -2 . The velocity of light is taken unity.

where \mathbf{A} is the initial acceleration at $t = 0$, describes runaway (or self-accelerated) motion. For $\mathbf{A} = 0$, we have $\mathbf{a} = 0$, and $\mathbf{v} = \text{const}$. Thus a free electron can behave as both Galilean ($\mathbf{A} = 0$), and non-Galilean ($\mathbf{A} \neq 0$) objects.

Where does the Abraham–Lorentz equation come from? The scheme of its derivation is as follows. We first solve Maxwell’s equations

$$\square A^\mu(x) = 4\pi e \int_{-\infty}^{\infty} ds v^\mu(s) \delta^4(x - z(s)) \quad (12)$$

where the world line of a single charge $z^\mu(s)$ is taken to be arbitrarily prescribed timelike smooth curve. The retarded Lienárd–Wiechert solution

$$A^\mu(x) = \frac{e v^\mu}{(x - z) \cdot v} \Big|_{s=s_{\text{ret}}} \quad (13)$$

is regularized and substituted into the equation of motion for a bare charged particle

$$m_0 \mathbf{a} = e (\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (14)$$

where m_0 is the bare mass. We then require that the renormalized mass

$$m = \lim_{\epsilon \rightarrow 0} \left(m_0(\epsilon) + \frac{e^2}{2\epsilon} \right) \quad (15)$$

be a finite positive quantity. Finally, we arrive at the Abraham–Lorentz equation (8) in the limit of the regularization removal $\epsilon \rightarrow 0$.

In order to derive the equation of motion for a massive particle capable of emitting gravitational waves, one should repeat the essentials of this procedure: find the retarded solution to Einstein’s equations (2) with $\Lambda = 0$ assuming that a given point particle which generates the retarded gravitational field moves along an arbitrarily prescribed timelike smooth world line, regularize this solution, substitute it into the equation of motion for the bare particle, perform the mass renormalization, and remove the regularization. This will yield the desired equation of motion for a dressed massive particle, which is apparently different from the equation of a geodesic

$$\frac{dv^\lambda}{ds} + \Gamma_{\mu\nu}^\lambda v^\mu v^\nu = 0 \quad (16)$$

where $\Gamma_{\lambda\mu\nu}$ is the Christoffel symbol for the background metric $g_{\mu\nu}$

$$\Gamma_{\lambda\mu\nu} = \frac{1}{2} (\partial_\lambda g_{\mu\nu} + \partial_\mu g_{\nu\lambda} + \partial_\nu g_{\lambda\mu}). \quad (17)$$

This project is highly nontrivial. Even the first stage of it has defied implementation: by now, no retarded solution to the gravitation equations (2) similar to the Lienárd–Wiechert solution (13) in electrodynamics is found. There is in addition a conceptual problem. While the kinematical criterion for availability of electromagnetic radiation is that the world line of the source is curved (which makes absolute geometric sense in the Maxwell–Lorentz theory formulated in Minkowski space), such a criterion is unsound for the gravitational field source, say, a field singularity in a smooth manifold, since all timelike world lines of this point source are regarded as diffeomorphic-equivalent in the framework of general relativity. A plausible resolution of this problem is to discriminate between geodesics for a given background metric and world lines deviating from these geodesics. It may be disappointing that we can not offer a complete explanation for the accelerated expansion of the Universe based on self-accelerated solutions analogous to solution (11) of the Abraham–Lorentz equation. Nevertheless, relying on results drawn from solvable theories in which particles interact with scalar, Yang–Mills, and linearized gravitational fields, together with dimensional considerations, we cope with a modest task: guess some essential features of the behavior of a particle emitting gravitation waves.

The paper is organized as follows. Section 2 demonstrates that, regardless of warnings, the self-accelerated solution (11) is consistent with every fundamental physical principle and experimental fact. In the next section, we take a brief look at the self-deceleration and the equations of motion for dressed particles interacting with the Yang–Mills, scalar, and tensor fields. In the final section we sketch the broad outline of the equation of motion for a dressed massive particle interacting with gravitation field and its relevance to the accelerated regime of the Universe development.

2 Dressed charged particles

Many people, being prejudiced against solution (11), accuse it of two sins: violation of energy conservation and lack of experimental evidence for self-accelerated motions. Our immediate task is to show that both accusations are unjust.

The generalization of the Abraham–Lorentz equation (8) to the special relativistic context is the Abraham–Lorentz–Dirac equation (see, e. g., [9]–[12])

$$ma^\lambda - \frac{2}{3} e^2 (\dot{a}^\lambda + v^\lambda a^2) = f^\lambda \quad (18)$$

where $a^\mu = \dot{v}^\mu$ is the four-acceleration. This equation accounts for the dynamics of a synthesized object whose inertia is characterized by the quantity m defined in Eq. (15) containing both mechanical and electromagnetic contributions. We will call this object the dressed charged particle. The state of the dressed particle is specified by the four-coordinate of the singular field point z^μ and the four-momentum

$$p^\mu = m v^\mu - \frac{2}{3} e^2 a^\mu \quad (19)$$

assigned to this point. (Teitelboim [13] was the first to derive this expression with the aid of some invariant regularization of the retarded Lienárd–Wiechert field.) The singular field point is governed by the equation

$$\overset{v}{\perp} (\dot{p} - f) = 0 \quad (20)$$

where $\overset{v}{\perp}$ is the operator

$$\overset{v}{\perp} = \mathbf{1} - \frac{v \otimes v}{v^2} \quad (21)$$

projecting vectors on the hyperplane perpendicular to the world line, p^μ is the four-momentum defined in (19), and f^μ is an external four-force applied to the point z^μ . Indeed, substitution of (19) in (20) gives the Abraham–Lorentz–Dirac equation (18). On the other hand, Eq. (20) is nothing but Newton’s second law in a coordinate-free form [7].

The Abraham–Lorentz equation (8) may also be cast as Newton’s second law in its original form

$$\frac{d\mathbf{p}}{dt} = \mathbf{f} \quad (22)$$

where \mathbf{p} is the three-momentum of a nonrelativistic dressed particle,

$$\mathbf{p} = m \mathbf{v} - \frac{2}{3} e^2 \mathbf{a}, \quad (23)$$

which is obtained from the space component of Eq. (19) in the limit $\mathbf{v} \rightarrow 0$.

Teitelboim [13] was able to show that the Abraham–Lorentz–Dirac equation (18) is equivalent to the local energy-momentum balance,

$$\dot{p}^\mu + \dot{\mathcal{P}}^\mu + \dot{\wp}^\mu = 0, \quad (24)$$

where the four-momentum of the dressed particle p^μ is defined in (19), the four-momentum rate of radiation emitted by the charge $\dot{\mathcal{P}}^\mu$ is represented by the Larmor formula,

$$\dot{\mathcal{P}}^\mu = -\frac{2}{3} e^2 v^\mu a^2, \quad (25)$$

and the external four-momentum rate $\dot{\wp}^\mu$ relates to the external Lorentz four-force,

$$\dot{\wp}^\mu = -f^\mu. \quad (26)$$

The balance equation (24) reads: the four-momentum extracted from the external field $d\wp^\mu = -f^\mu ds$ is spent on the variation of the four-momentum of the dressed particle dp^μ and the four-momentum $d\mathcal{P}^\mu$ carried away by the radiation.

With $f^\mu = 0$, Eq. (18) is satisfied by

$$v^\mu(s) = \alpha^\mu \cosh(w_0 \tau_0 e^{s/\tau_0}) + \beta^\mu \sinh(w_0 \tau_0 e^{s/\tau_0}) \quad (27)$$

where α^μ and β^μ are constant four-vectors that meet the conditions $\alpha \cdot \beta = 0$, $\alpha^2 = -\beta^2 = 1$, w_0 is an initial acceleration magnitude, and τ_0 is given by (10). The solution (27) describes a runaway motion, which degenerates to the Galilean regime when $w_0 = 0$.

We see from (11) and (27) that the class of Galilean world lines is distinct from the class of runaway world lines. A dressed particle may either show itself constantly as a Galilean object or execute perpetually a self-accelerated motion. It is impossible to render a Galilean dressed particles self-accelerated and vice versa. The non-Galilean behavior is an innate feature of some species of dressed particles. (It is remarkable that the Maxwell–Lorentz theory alone gives no way of deducing the distribution of dressed charged particles by these two classes. In fact, the class of self-accelerated dressed particles may well be empty for some unknown reason.)

This observation makes it clear that the mere existence of runaways does not contradict to Newton’s first law (‘every particle continues in its state of rest or uniform motion in a straight line unless it is acted upon by some exterior force’) since the essence of this law is to forbid the Galilean regime from changing into another regime at a finite instant.

It is often claimed that the solution (27) is nonphysical since the picture where a free particle continually accelerates and continually radiates seems contrary to energy conservation. (Notice that a ‘particle’ possessing the four-momentum $p^\mu = mv^\mu$, with its time component $p^0 = m\gamma$ being positive definite, is usually meant in this claim.) The energy of both this ‘particle’ and the electromagnetic field increases for no apparent reason.

It is the idea of a ‘particle’ with such a four-momentum which is deceitful and opens up numerous puzzles and paradoxes [14]. Indeed, while on the subject of what is governed by the Abraham–Lorentz–Dirac equation (18), one usually imagines an aggregate composed of a point charge and a Coulomb-like field train dragged behind it. The ascription of the four-momentum $p^\mu = mv^\mu$ to this aggregate lacks support from a careful analysis based on the appropriate definition of the radiation and invariant regularization procedures [13].

By contrast, proceeding from the concept of a dressed particle with the four-momentum defined in (19), we have the balance equation (24). Hence, there is no contradiction with energy conservation: in the absence of external fields, the energy variation of the dressed particle dp^0 is equal to the energy carried away by the radiation $-\dot{\mathcal{P}}^0 ds$. A subtlety is that the energy of the dressed particle

$$p^0 = m\gamma (1 - \tau_0 \gamma^3 \mathbf{a} \cdot \mathbf{v}) \quad (28)$$

is not positive definite. The indefiniteness of expression (28) means that the increase of velocity need not be accompanied by the increase of energy. It would, therefore, make no sense to inquire from where the particle extracts energy to accelerate itself. The energy of the self-accelerated dressed particle is actually diminished.

The fact that the energy of a dressed particle is indefinite might appear at first sight strange and counter-intuitive. But it is scarcely surprising if we recall the synthetic origin of the dressed particle, and note that the two contributions to m , defined in (15), are opposite in sign.

If $m = 0$, which is another reasonable option of the mass renormalization (15), then the first term of Eq. (18) disappears, and, with $f^\mu = 0$, it reduces to

$$(\perp \dot{a})^\mu = 0.$$

This is the equation of relativistic uniformly accelerated motion [11]. Therefore, the world line of a dressed particle with $m = 0$ in the absence of external forces is a hyperbola

$$v^\mu(s) = \alpha^\mu \cosh w_0 s + \beta^\mu \sinh w_0 s, \quad \alpha \cdot \beta = 0, \quad \alpha^2 = -\beta^2 = 1. \quad (29)$$

The constant curvature w_0 of such a world line may be arbitrary, in particular $w_0 = 0$.

It follows from (19) that

$$M^2 = p^2 = m^2 (1 + \tau_0^2 a^2). \quad (30)$$

If $\tau_0^2 a^2 < -1$, then the dressed particle turns to the tachyonic state, by which is meant a state with $p^2 < 0$. (We point out that our consideration is restricted to timelike smooth world lines, and the tachyonic state under discussion is unrelated to superluminal motions; more precisely, the advent of spacelike four-momenta is peculiar to sufficiently large curvatures of the world line, rather than spacelike run of it.)

Let a dressed particle be moving in the runaway regime (11). Then, after a lapse of time

$$\Delta t = -\tau_0 \log \tau_0 |\mathbf{A}|, \quad (31)$$

the critical acceleration $|\mathbf{a}| = \tau_0^{-1}$ will be exceeded, and the four-momentum of this dressed particle will be spacelike.

This result provides an explanation for the fact that self-accelerated dressed charged particles, if any, were never observed. The period of time over which a self-accelerated electron possesses timelike four-momenta is quite tiny. From (31) and (10), the period Δt is estimated at $\tau_0 \sim 10^{-23}$ s for electrons, and still shorter for more massive charged elementary particles. All primordial self-accelerated particles with such τ_0 's have long been in the tachyonic state. However, we have not slightest notion of how tachyons can be experimentally recorded. (It seems plausible that self-accelerated particles, transmuted into tachyons, represent part of dark matter.)

If a cosmological object is considered as a dressed particle emitting gravitational waves, the characteristic period τ_0 may be found to be comparable with the inverse current Hubble scale H^{-1} . The self-acceleration of such an object can indeed be observed at the present time. For clarity, the experimental value of this scale is $H^{-1} = (46 \pm 4) \cdot 10^{16}$ s.

3 Self-deceleration

Consider a dressed colored particle ('quark' for short) in the cold QCD phase [15]. The emission of Yang-Mills waves by an accelerated quark in this phase is attended with energy gains, rather than energy losses. Accordingly, the equation of motion for a dressed quark with the color charge Q in an external Yang-Mills field $F^{\mu\nu}$ is

$$m [a^\mu + \ell (\dot{a}^\mu + v^\mu a^2)] = \text{tr}(Q F^{\mu\nu}) v_\nu \quad (32)$$

where m is the renormalized mass, ℓ is the characteristic period,

$$\ell = \frac{8}{3mg^2} \left(1 - \frac{1}{\mathcal{N}}\right), \quad (33)$$

g is the Yang-Mills coupling constant, and $\mathcal{N} \geq 2$ is the number of colors.

Although both equations (32) and (18) contain the so-called Abraham term

$$\Gamma^\mu = \dot{a}^\mu + v^\mu a^2, \quad (34)$$

they show marked distinction in that Γ is multiplied by coefficients of opposite sign.

For $F^{\mu\nu} = 0$, the general solution to Eq. (32)

$$v^\mu(s) = \alpha^\mu \cosh(w_0 \ell e^{-s/\ell}) + \beta^\mu \sinh(w_0 \ell e^{-s/\ell}), \quad (35)$$

with $\alpha \cdot \beta = 0$, $\alpha^2 = -\beta^2 = 1$, describes self-decelerated motion.

By contrast, in the hot QCD phase (where the Yang-Mills-Wong system is Abelian, and hence linearized [15]), the equation of motion for a dressed quark is similar to (18), and, therefore, dressed quarks execute self-accelerated motions like those given by (27).

At first sight, the self-deceleration is quite innocuous phenomenon, because the motion becomes almost indistinguishable from Galilean in the short run. However, the presence of self-decelerations does jeopardize the consistency of the theory. Indeed, as we go to the past, the acceleration increases exponentially, and the rate of the energy gain grows along with it. Thus, the energy of the Yang-Mills field at any finite instant is divergent. On the other hand, the self-acceleration does not play such a fatal role in electrodynamics with the retarded boundary condition (and also the hot QCD phase); this

non-Galilean regime of motion does not entail ‘infrared’ divergences. Indeed, substitution of (27) in (25) shows that the electromagnetic field energy radiated by a self-accelerated particle during a half-infinite period of time is finite.

A plausible resolution of the trouble with ‘infrared’ divergences due to a non-Galilean evolution of dressed quarks in the Yang–Mills theory³ is to impose the supplementary condition that the acceleration of dressed quarks be always less than the critical one, $|a| \leq \ell^{-1}$, in other words, the curvature of the allowable world lines must not exceed ℓ^{-1} . If, for some reason, the critical acceleration is yet attained in the cold phase, then the dressed quark must plunge into the hot phase, rather than turn to the tachyonic state. Thus, the interconversion from one phase to another is a means to circumvent the difficulty with infinite energy storage in some nonlinear field theories.

Let us briefly run through the non-Galilean behavior of dressed particles interacting with massless scalar and tensor fields (for a review of the Lagrangian description of bare particles coupled with these fields see, e. g., [10]). The equation of motion for a dressed particle in an external scalar field ϕ is [16]

$$\frac{d}{ds} (m + g\phi) v^\mu - \frac{1}{3} g^2 (\dot{a}^\mu + a^2 v^\mu) = g \partial^\mu \phi \quad (36)$$

where m is the renormalized mass, and g is the coupling constant. The coefficient of the Abraham term in this equation is of the same sign as that in (18), and so the non-Galilean behavior for the object governed by Eq. (36) can be classified as self-accelerated motion.

There are several versions of the interaction of a dressed particle with a massless symmetric tensor field $\phi_{\alpha\beta}$ [10, 16], among which one, being a combination of interactions with scalar and tensor fields known as the ‘linearized gravity’, holds the greatest interest for our discussion. The equation of motion for a dressed particle in an external tensor field $\phi_{\alpha\beta}$ of the linearized gravity with the retarded boundary condition is [17, 16]

$$\begin{aligned} \frac{d}{ds} \left[v^\mu \left(1 - \frac{1}{2} v^\alpha v^\beta \phi_{\alpha\beta} - \frac{1}{2} \phi^\alpha_\alpha \right) + v_\alpha \phi^{\alpha\mu} \right] + \frac{11}{3} G m (\dot{a}^\mu + a^2 v^\mu) \\ = \frac{1}{2} v^\alpha v^\beta \partial^\mu \phi_{\alpha\beta} - \frac{1}{4} \partial^\mu \phi^\alpha_\alpha \end{aligned} \quad (37)$$

where G is the gravitational constant, and m is the renormalized mass. The coefficient of the Abraham term in this equation is of the same sign as that in (32), which implies that the non-Galilean regime for this dressed particle refers to self-decelerated motion. While on the subject of the linearized gravity in Minkowski space as a field theory on its own right, not an approximation to general relativity, it should presumably be thought of as inconsistent, because the system is linear (unlike the Yang–Mills system), and, therefore, reveals a single phase.

The above results concerning the equations of motion for different dressed particles are summarized in the following table

Table 1. Abraham term in different theories

Scalar field	Abelian vector field	Yang–Mills field (cold phase)	Linearized gravity
$-\frac{1}{3} g^2 \Gamma^\mu$	$-\frac{2}{3} e^2 \Gamma^\mu$	$\frac{2}{3} Q^2 \Gamma^\mu$	$\frac{11}{3} G m^2 \Gamma^\mu$

4 Discussion

Everybody, who is confident of the physical reality of gravitational radiation⁴, should recognize that the equation of motion for a massive particle emitting gravitational waves is different from the equation

³The above conclusion regarding the existence of two impermeable classes of dressed particles, Galilean and non-Galilean, holds for the present discussion. These classes are populated according to requirements which are beyond the control of the Yang–Mills–Wong theory. It is possible that some yet-to-be-known requirement for the class of self-decelerated quarks is that it be empty.

⁴Infeld [19] argued that gravitational radiation does not occur at all. However, his consideration is based on the approximation method of Einstein, Infeld, and Hoffmann, invoking peculiar assumptions,

of a geodesic (16). For dimensional reasons, the desired equation must include, apart from terms of equation (16), some higher-derivative terms, a diffeomorphic-covariant generalization of the Abraham term⁵, which would result in an additional solution distinct from that describing the geodesic for the background metric⁶.

As discussed in Section 3, the coefficient of the Abraham term in the linearized gravity is positive, hence a massive dressed particle seems to move in a self-decelerated fashion. This result is adverse to the suggested explanation for the accelerated expansion of the Universe.

Remember, however, that the linearized gravity is not a well-defined approximation to general relativity. In fact, there is no unambiguous approximation scheme which is, in its physical outcome, independent of coordinate and boundary conditions. For example, the famous Einstein–Infeld–Hoffmann approximation operates on the premise that the time derivative of any field quantity is much smaller than the spatial derivatives, which has proved itself in nonrelativistic problems. With this approximation, the gravitational dipole moment of a dressed particle about the center-of-mass vanishes (together with its gravitational radiation), and the derivatives of acceleration in the resulting equation of motion are cancelled, leaving room for higher order derivatives.

An alternative approximation procedure [17], which treats time and space coordinates on the same footing, and utilize the retarded boundary condition, results in the Lorentz-invariant equation of motion for a dressed particle (37). However, terms of the same order as the Abraham term may appear implicitly in (37) through the retardation effect in the metric; they are of opposite sign and cancel the main contribution to the Abraham term.

It is well to bear in mind that the runaway solution (11) contains $\exp(3mt/2e^2)$, that is, reveals an essential singularity at $e = 0$. This implies that perturbative treatments of Einstein’s equations, using ‘smallness’ of the gravitation coupling constant G , may be found to be unsuitable for the analysis of nonperturbative effects, non-geodesic motions in particular.

On the other hand, there is good indirect evidence that the pulsar PSR 1913+16, which is a constituent of a binary neutron star system, loses (rather than gains) energy for its gravitational emission, rendering the observed rotation period of this system gradually decreasing [20]. This result lends support to our assumption that the actual deviation from a geodesic is represented by a self-acceleration.

Let us estimate the characteristic time $\tau_0 = \frac{11}{3}Gm$ in (37). Taking m to be a typical cluster mass, we have $\tau_0 \sim 10^8 \text{ s} \sim 3 \text{ years}$, which is very far from the desired value $\tau_0 \sim H^{-1} \sim 10^{10} \text{ years}$.

One may hope that some way out would be found if, e. g., the interactions with other fields (electromagnetic, dilaton, gluon, etc.) would be appropriately combined with gravitational interaction. Contributions of different signs from these interactions may have a dramatic effect on the coefficient of the aggregate Abraham term. In addition, one should keep in mind that the representations of galaxies and clusters as simple poles of the gravitational field may be not adequate, since these objects are endowed with intrinsic angular momentum. However, the description of a radiating charged particle with spin is a challenging problem (for a review see [21]), discarded here, let alone the description of a spinning massive particle emitting gravitational waves.

However, the most striking possibility resides in the fact that the characteristic time $\tau_0 = Gm$ with m being the total visible mass of the Universe is of order of the inverse current Hubble scale H^{-1} . Does this mean that the Universe as a whole executes self-accelerated motion? Should this be the case, then a pronounced anisotropy in the velocity distribution of distant supernovae would be an observable consequence.

Acknowledgment

I would like to thank I. D. Novikov for helpful comments. This work was supported in part by the International Science and Technology Center under the Project # 840.

and his findings, being sensitive to a particular choice of coordinate and boundary conditions, are open to argument.

⁵Conceivably, this generalization is given by a complicated integro-differential expression similar to that for a dressed charged particle in a curved space [18].

⁶Again, one may expect the existence of two disjoint classes of dressed particles, geodesic and non-geodesic. It may well be that some unknown fundamental principle, unrelated to the gravitation theory, requires that the class of non-geodesic particles be empty. We assume that such is not the case.

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